靜宜大學 103 學年度碩士班招生考試試題

學系：財務與計算數學系　科目：線性代數

1. (10 %) Let $\vec{u} = \begin{bmatrix} 2 \\ 1 \\ -2 \\ 5 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} -1 \\ 0 \\ 3 \\ -2 \end{bmatrix}$. Determine if $\vec{x} = \begin{bmatrix} 4 \\ 3 \\ 0 \\ 11 \end{bmatrix}$ lies in the subspace $U = \text{span}\{\vec{u}, \vec{v}\}$.

2. (8 %) If A is a 5×5 matrix with $\det(A) = 3$, compute $\det(-2A)$.
(7 %) If 4 of 5 eigenvalues of A are: -1, $\frac{1}{5}$, $\frac{1}{7}$, and 1. Find the fifth eigenvalue of A.

3. (10 %) Find **all** solutions to the following system of linear equations.

$$\begin{cases} x_1 - 4x_2 - x_3 + x_4 = 3, \\ 2x_1 - 8x_2 + x_3 - 4x_4 = 9, \\ -x_1 + 4x_2 - 2x_3 + 5x_4 = -6. \end{cases}$$

4. (20 %) Find bases for the row and column spaces of $A = \begin{bmatrix} -1 & 2 & 0 & 2 \\ 3 & -6 & 1 & 10 \\ 1 & -2 & 1 & 14 \end{bmatrix}$
(5 %) Determine the rank of A.

5. (10 %) Find a 2×2 real matrix A that has an eigenvalue $\lambda_1 = 1$ with eigenvector $\vec{e_1} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and an eigenvalue $\lambda_2 = -1$ with eigenvector $\vec{e_2} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.
(10 %) Compute $A^{100} + A$.

6. (10 %) If A is an $m \times m$ matrix and $(A-I)(A-2I)(A-3I) = O$, where O is the $m \times m$ zero matrix. Show that the only possible eigenvalues of A are 1, 2, and 3.

7. (10 %) If matrix A is $m \times n$ and matrix B is $n \times m$, show that $AB = O$ if and only if $\text{col}(B) \subseteq \text{null}(A)$. ($O$ is the $m \times m$ zero matrix.)