# 靜宜大學 106 學年度碩博士班暨碩士在職專班招生考試試題

學系：財務與計算數學系　　　　　　科目：線性代數

1. Consider the following system of equations:

$$\begin{cases} x - y - 3z = 3, \\ 2x + z = 0, \\ 2y + 7z = c. \end{cases}$$

(a) (10 points) For what values of $c$ does the system have a solution?

(b) (10 points) For the value of $c$ you found in (a), what are the solutions of the system of equations?

2. Let $A = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 0 & -3 \\ 1 & 1 & 2 \end{bmatrix}$, and $C = \begin{bmatrix} 3 & 1 & 1 \\ -1 & -2 & 0 \end{bmatrix}$.

(a) (10 points) Compute $A(2B + 3C)$.

(b) (10 points) Compute $B^t A^t$, where $A^t$ is the transpose of $A$.

3. Let $A$ be the matrix $\begin{bmatrix} 0 & -1 & 1 \\ 2 & 3 & 7 \\ 1 & 3 & 2 \end{bmatrix}$.

(a) (5 points) Compute $\mathrm{tr}(A)$, the trace of the matrix $A$.

(b) (5 points) Compute $\det(A)$, the determinant of the matrix $A$.

(c) (10 points) Compute $\mathrm{rank}(A)$, the rank of the matrix $A$.

4. Suppose that $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation, $T(1,0) = (1,4)$, and $T(1,1) = (2,5)$.

(a) (10 points) What is $T(2,3)$?

(b) (10 points) Is $T$ one-to-one?

5. (a) (10 points) Find a $2 \times 2$ real matrix A that has an eigenvalue $\lambda_1 = 1$ with eigenvector $\vec{u_1} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ and an eigenvalue $\lambda_2 = -1$ with eigenvector $\vec{u_2} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$.

(b) (10 points) Compute $A^{100} + A$.